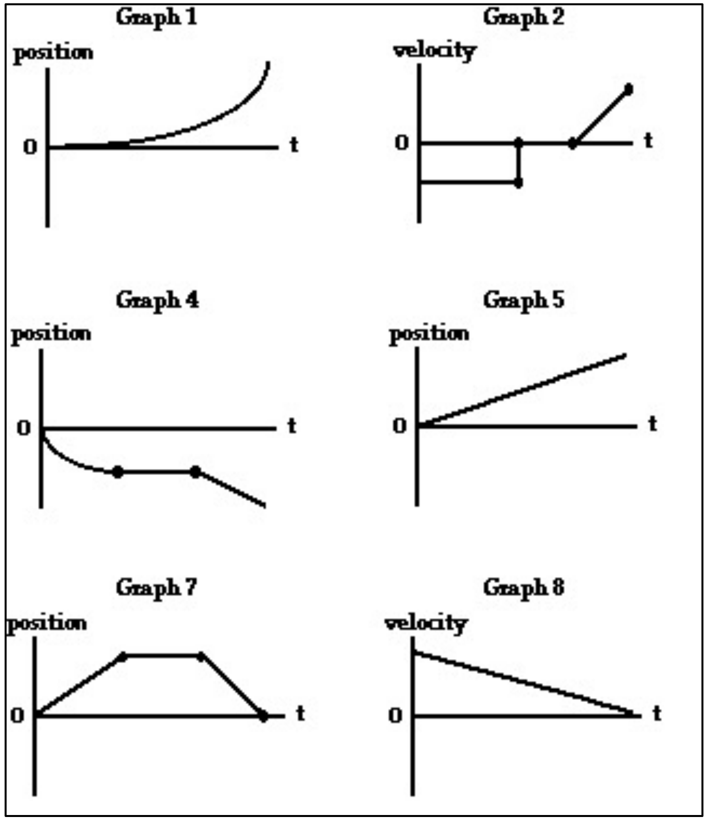
How are you doing with graphs of motion?

- Which graph(s) to the right best match the following descriptions?

- A. An object moves with constant velocity. **Graph #5**
- B. An object moves backwards, then stops suddenly, then begins to speed up moving forward. **Graph #2**
- C. An object moves with positive acceleration. **Graph #1**
- D. An object moves forwards, stops suddenly, then moves backwards to its initial spot. **Graph #7**
- E. An object decelerates backwards, stops, then moves backwards at a constant velocity. **Graph #4**



General announcements

- Turn in hmwk
- Questions on expectations, calendar, or class Website?
- Today: intro to kinematics and graphs
- Tomorrow: first lab! We’ll talk about write-ups etc. (this is a short one)
- Big Note: If you email me for anything, please include in the message THE CLASS PERIOD you are in. Otherwise, I have to scramble to figure out where you belong, that that just takes additional time.
- Reminder: if you receive accommodations on tests, come talk to me so we can make a plan
  - Test 1 is Thursday, 9/5

Island Series Introduction

Island series 1: Acceleration walk

- How can one student in your group <u>accelerate at a constant rate</u> for 10 meters?

  – Parameters:

    - You will only get <u>one</u> shot to actually try this! You need to work together to come up with a clear procedure that anyone in the group can follow.
    - Talk through potential challenges or pitfalls before settling on a procedure.
    - We are talking about a non-zero acceleration here!

Following up…

- What would we need to do or know in order to successfully complete the island challenge?

- What does acceleration mean…
  - Conceptually?

  - Mathematically?

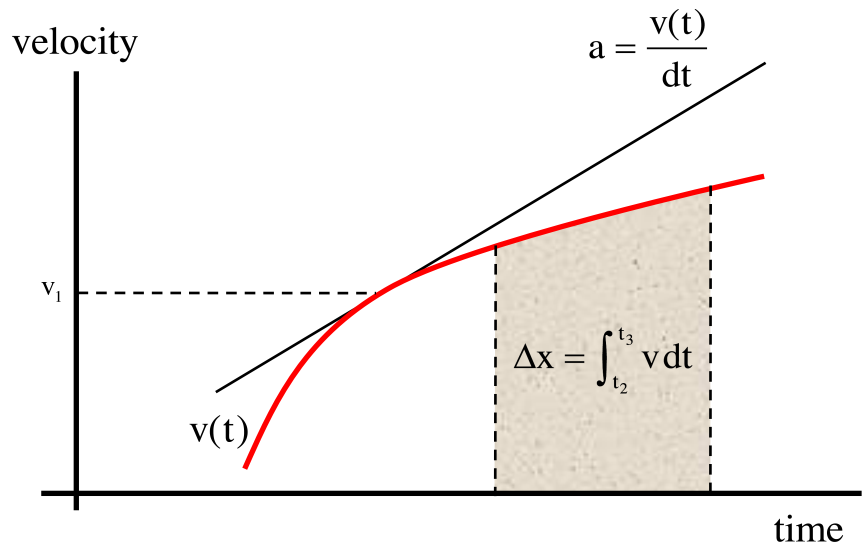
Mathematical relationships

- *Looking at* the graph to the right:
  - The *slope of the tangent line to a velocity vs. time graph* gives the rate of **change of velocity**…aka ***acceleration*****:**

$$a = \frac{dv}{dt}$$

  - The *area under a velocity vs. time graph* yields the **net displacement** of the object**:**

$$\Delta x = \int_{t_1}^{t_2} v\,dt$$



Mathematical relationships

- What about **position vs. time**?

- Sketch a graph of position vs. time for an object that has:
  - No velocity
  - A constant velocity
  - Changing velocity

- For a **position vs. time** graph, a **constant slope = constant velocity**. The **slope of the tangent line** at any point gives the **instantaneous velocity** of the object.

Graphical/mathematical relationships summed up

- So, for a **velocity vs. time** graph, the
  - **SLOPE** of the tangent line = **acceleration**
  - **AREA** under the graph = **displacement**

- What about for a **position vs. time** graph?
  - The **SLOPE** represents: **velocity**
  - If the graph is curved, what does that mean?

Changing slope means changing velocity – object is accelerating

- What about for an **acceleration vs. time** graph?
  - **AREA** under the graph **= velocity change**
  - **SLOPE** of the tangent line = "jerk" (not often used)

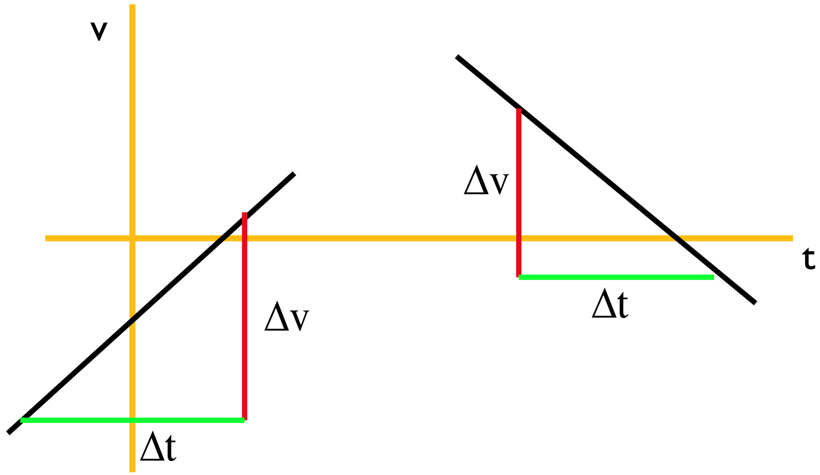
Kinematics derivations: the velocity equation

- *Acceleration* is equal to the slope of a velocity vs. time graph.
- *If* that acceleration is constant, the slope becomes:

$$a = \frac{\text{rise}}{\text{run}} = \frac{\Delta v}{\Delta t}$$

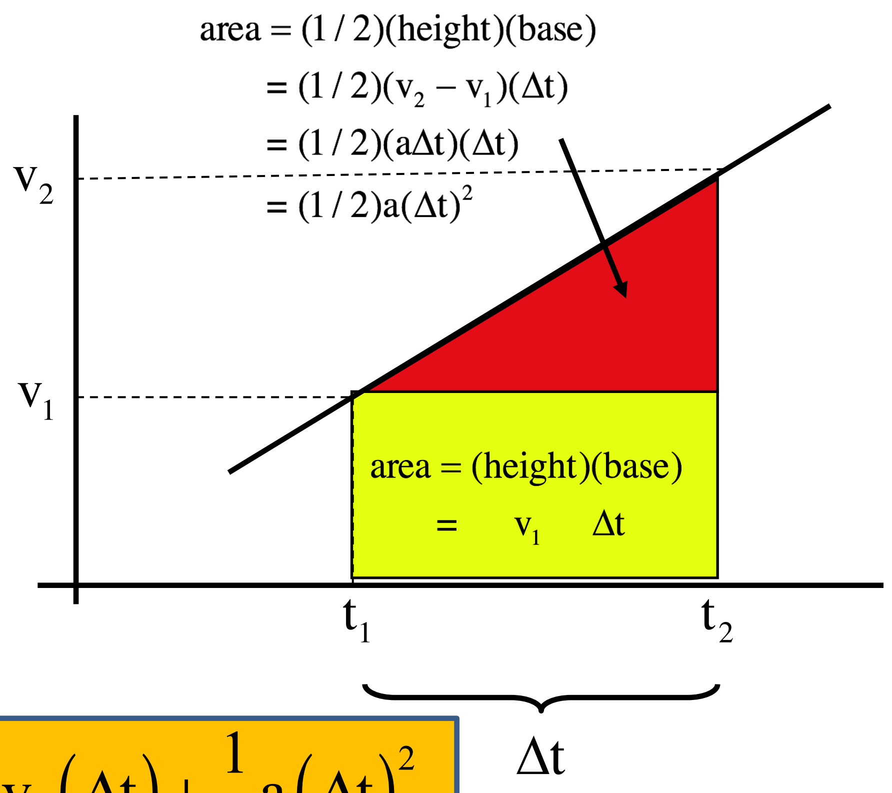
- *This* can be rewritten two ways:

$$a = \frac{v_2 - v_1}{\Delta t} \quad \text{or} \quad v_2 = v_1 + a\Delta t$$



Kinematics derivations: the displacement equation

- *Displacement* is equal to the area underneath a velocity vs. time graph.
- *If* the acceleration is constant, the graph is a straight line. To find the area



- *This* can be rewritten two ways:

$$\Delta x = v_1(\Delta t) + \frac{1}{2}a(\Delta t)^2 \quad \text{or} \quad x_2 = x_1 + v_1(\Delta t) + \frac{1}{2}a(\Delta t)^2$$

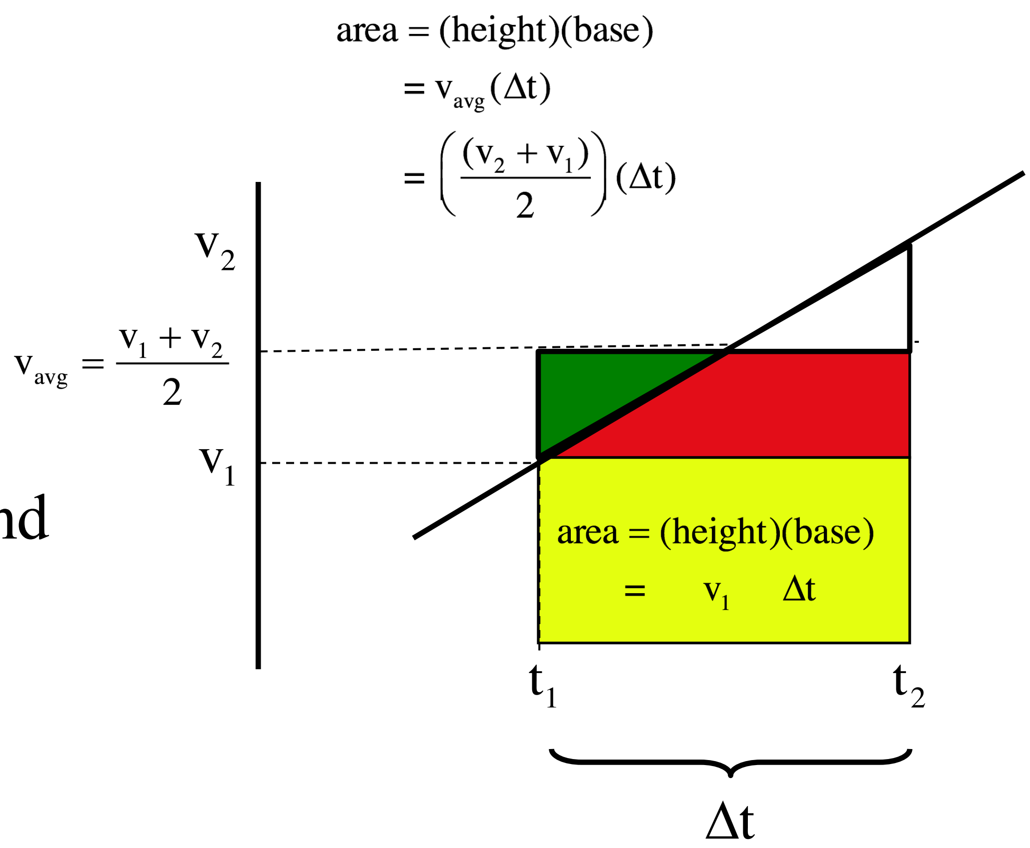
Kinematics derivations: the average-velocity equations

- For a constant acceleration situation:

$$v_{avg} = \frac{v_2 + v_1}{2}$$

- The area under the graph is still the net displacement, and using v_{avg}:

$$\Delta x_{net} = v_{avg}\Delta t$$



Kinematics derivations: the combination equation

Note: you will never have to derive this for me, but you should know where it comes from!

We know from before that $v_{avg} = \frac{v_2 + v_1}{2}$ and $\Delta x_{net} = v_{avg}\Delta t$ for any constant acceleration situation

We are going to combine these with the definition of acceleration $a = \frac{v_2 - v_1}{\Delta t}$ to derive the third kinematic equation.

From $\Delta x_{net} = v_{avg}\Delta t$ and v_{avg} we can write:

$$\Delta t = \frac{\Delta x}{v_{avg}}$$

$$= \frac{\Delta x}{\left((v_2 + v_1) / 2 \right)}$$

Kinematics derivations: the combination equation

Substituting this into $a = \frac{v_2 - v_1}{\Delta t}$ yields:

$$a = \frac{(v_2 - v_1)}{\frac{\Delta x}{\left((v_2 + v_1)/2\right)}} = \frac{(v_2 - v_1)(v_2 + v_1)}{2\Delta x}$$

$$\Rightarrow \quad 2a\Delta x = (v_2)^2 - (v_1)^2$$

$$\Rightarrow \quad \boxed{(v_2)^2 = (v_1)^2 + 2a\Delta x}$$

In college classes, the derivations start with the fact that the acceleration is constant and use Calculus to derive the equations. YOU WILL NEVER HAVE TO DUPLICATE THIS--it's solely for your own edification and amusement.

$$a = \frac{dv}{dt}$$

$$\Rightarrow \quad a\,dt = dv$$

$$\Rightarrow \quad a\int_{t_1}^{t_2} dt = \int_{v_1}^{v_2} dv$$

$$\Rightarrow \quad a(t_2 - t_1) = (v_2 - v_1)$$

$$\Rightarrow \quad a\Delta t = v_2 - v_1$$

$$\Rightarrow \quad v_2 = v_1 + a\Delta t$$

Noting that if the initial time is t = 0 (i.e., the clock starts when x_1 the object passes) and the final time is just "t," and with we denote the final velocity as just "v," the previous equations $v_2 = v_1 + a\Delta t$ goes to:

$$v = v_1 + at \qquad (\text{as } t - 0 = t)$$

With that, we can write:

$$v = \frac{dx}{dt} = v_1 + at$$

$$\Rightarrow \quad dx = v_1 dt + (at)dt$$

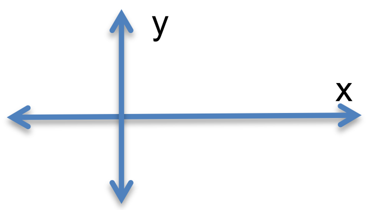
$$\Rightarrow \quad \int_{x_2}^{x_1} dx = v_1 \int_{t=0}^{t} dt + a\int_{t=0}^{t} t\,dt$$

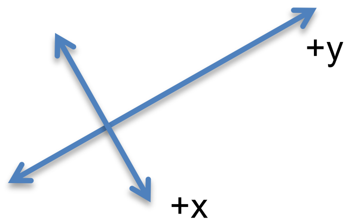
$$\Rightarrow \quad \Delta x = v_1 t + \frac{1}{2}at^2$$

$$\Rightarrow \quad x_2 = x_1 + v_1 \Delta t + \frac{1}{2}a(\Delta t)^2$$

Using kinematic equations: step 1

- Choose a frame of reference:
  - When approaching a problem, the first thing to do is pick a frame of reference (often, but not always, the stationary Earth is a good reference frame)
  - Once you've chosen a reference frame, stick with it! Don't change partway through – otherwise, your work will turn into gibberish.
  - Indicate your frame of reference with a **coordinate system.**
    - Indicate both the zero position <u>and</u> the positive directions





Which of these is better?

Why does it matter?

- Remember scalars vs. vectors?
  - Direction is very important for vectors – always measured relative to your coordinate system
- **Scalars** can be positive or negative, but don’t have direction
  - For example, the freeway speed limit is 65 mph, or the temperature of a room is 72 degrees Fahrenheit
- **Vectors** can be positive or negative depending on their direction
  - For example, my car’s velocity can be +65 mph or -65 mph depending on which direction I’m driving.

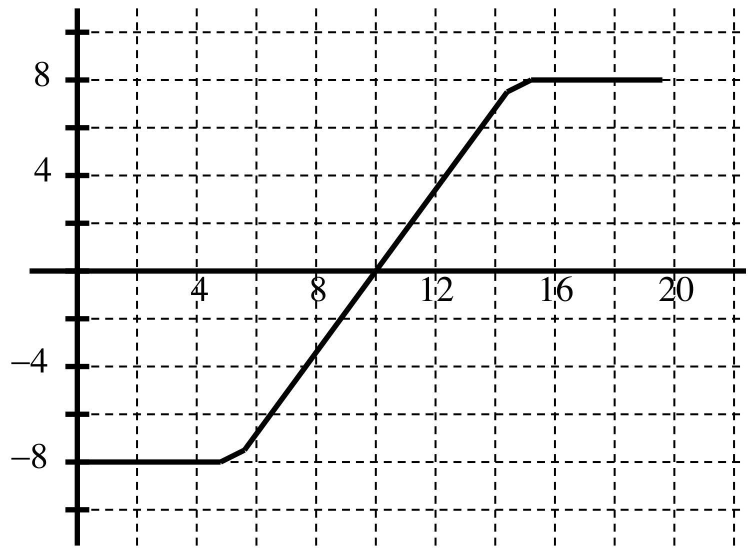
Figure 2.24 on p51

- What does this graph represent if it is a:

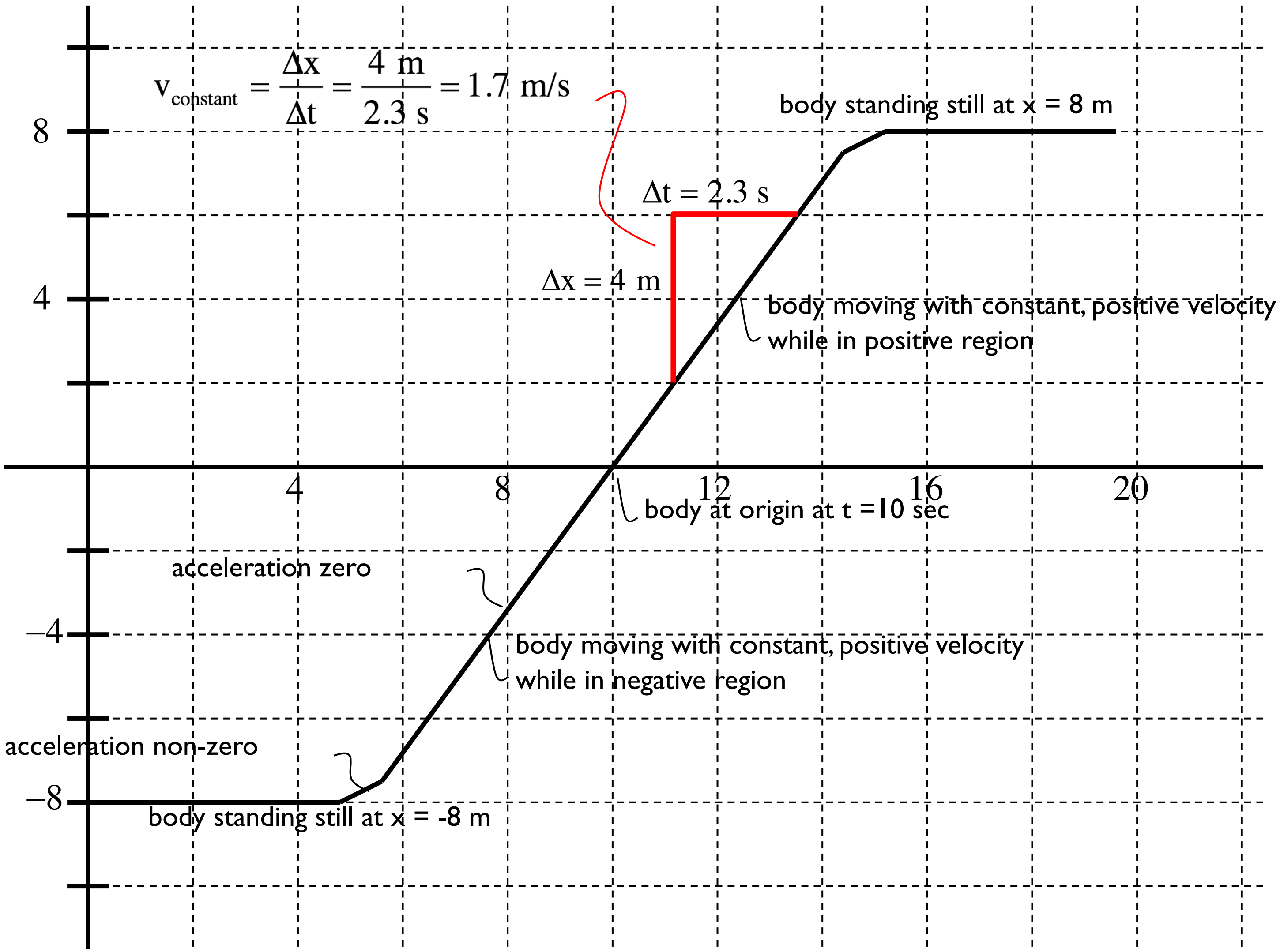
- Position vs. time graph?

- Velocity vs. time graph?

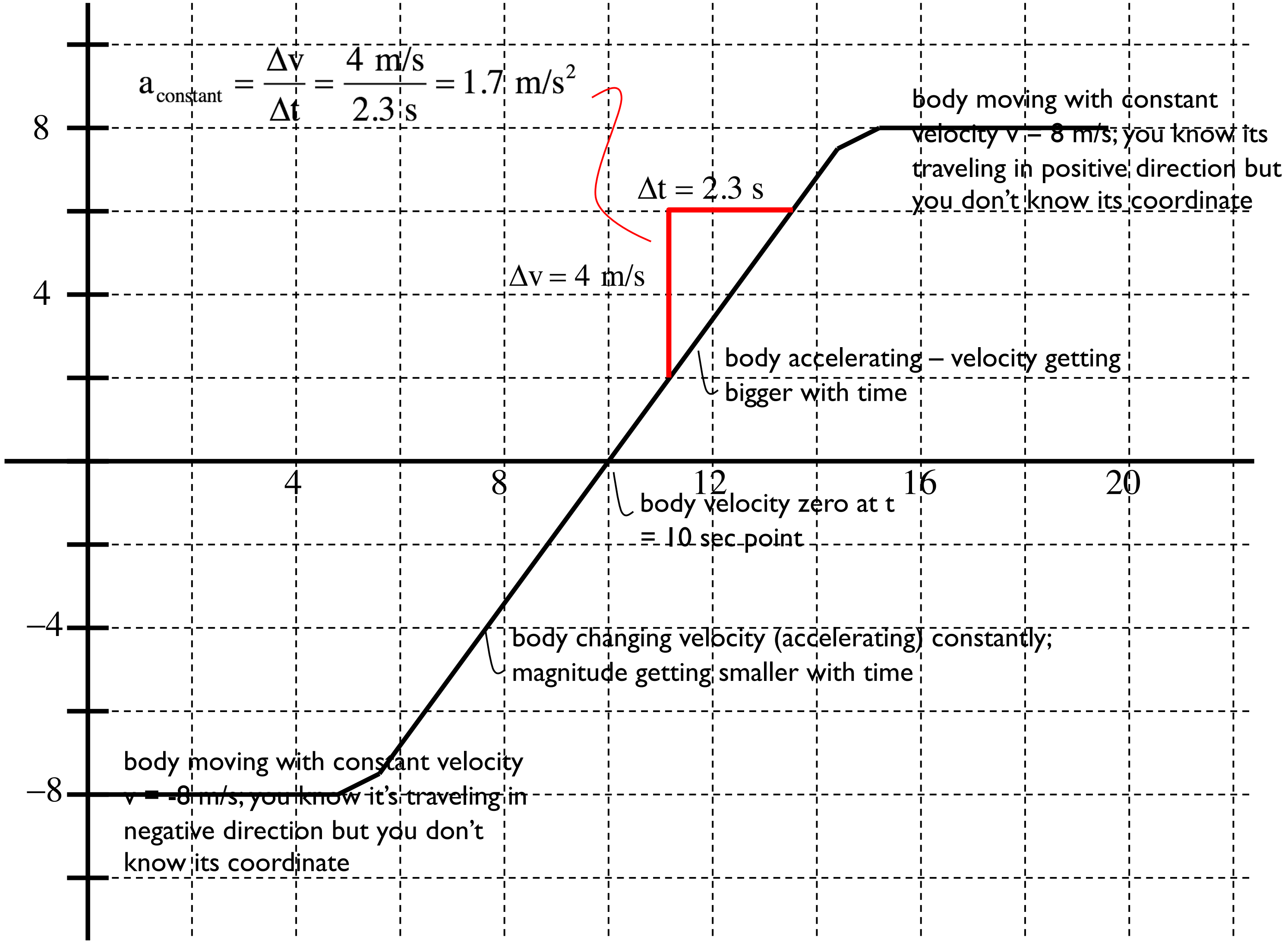
- Acceleration vs. time graph?



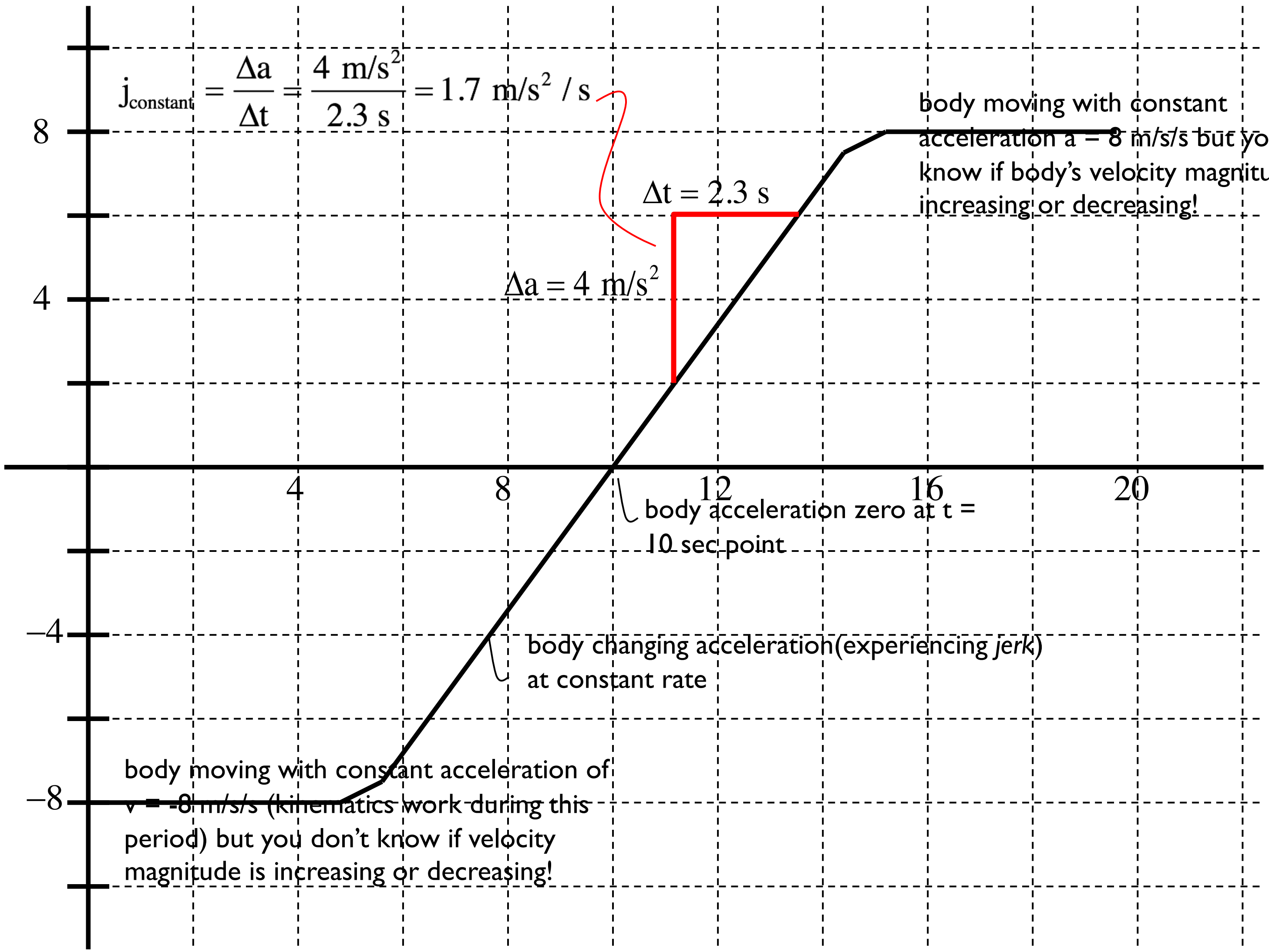
1.) As *position versus time* graph:



1.) As *velocity versus time* graph:



1.) As *acceleration versus time* graph:



Sign conventions

- The sign of a vector quantity indicates its **direction**.
- This requires that you clearly indicate your coordinate axes!
- What does it mean to have:
  - A positive velocity but negative position?
  - Negative displacement and a positive position?
  - A positive velocity and positive acceleration?
  - Negative velocity but positive acceleration?